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THE PROBABILITY OF HITTING AN AIRPLANE AS DEPENDENT UPON ERRORS IN THE HEIGHT FINDER AND THE DIRECTOR

by

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THE PROBABILITY OF HITTING AN AIRPLANE AS DEPENDENT UPON ERRORS IN THE HEIGHT FINDER AND THE DIRECTOR

Abstract

It is shown that the probability is 1/8 that the predicted future position of the airplane will lie in a volume surrounding the true future position and having a content which varies as the square of the time of flight and the product of the errors in the two angular rates provided by the director. From an analysis of the performance of service directors it is found that for fire against modern planes at high altitudes, the dimensions of this volume are large compared with the corresponding components of the dispersion of the points of burst and that therefore the probability of hitting varies inversely as this volume.

It is pointed out that there is an urgent need for directors of greater precision to augment the probability of hitting as well as for more accurate height finders.

The purpose of this report is to make a study of the combined effects of the errors of the height finder and the director on the probability of hitting an airplane. In making this study the conditions most favorable to fire control are assumed; the airplane is assumed to be moving at constant altitude with a constant speed and direction.

The function of the height finder is to measure the slant range, S, to the present position of the airplane, i.e., the slant range at the time the gun goes off (see figure 1).

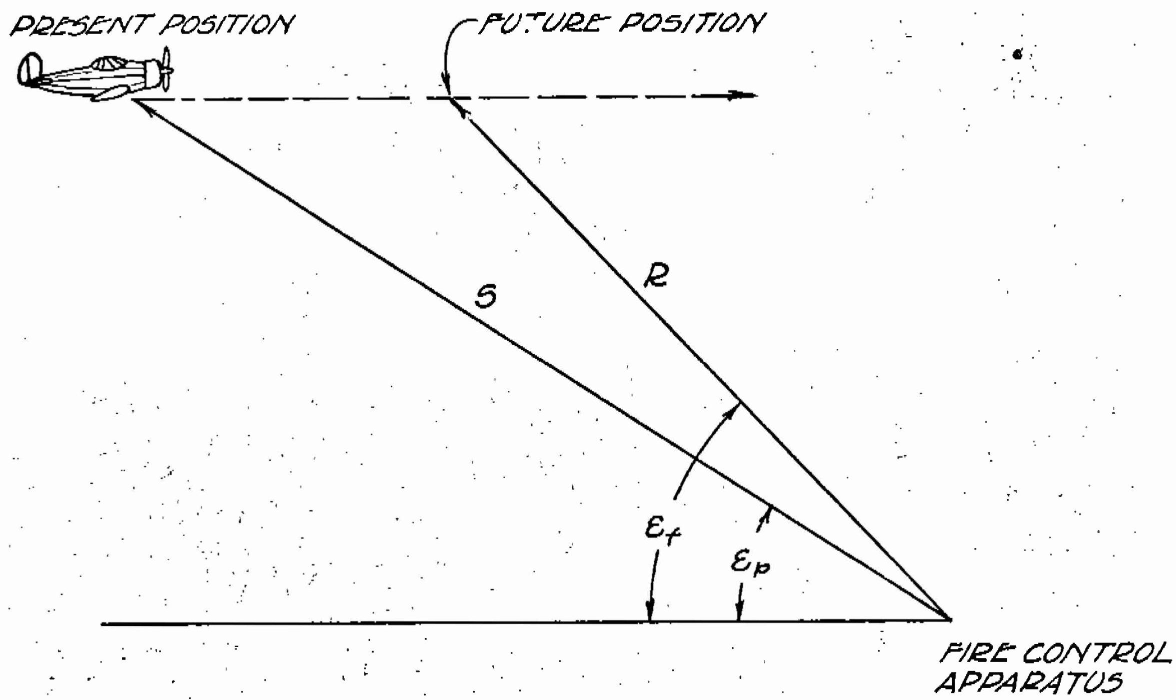

Fig. 1

For simplicity we consider for the present the airplane as moving directly toward the apparatus so that the azimuth and its rate of change are zero. Under these circumstances the director measures ε_p the present angle of elevation of the plane and the rate of change of ε_p which will be designated by ω_ε. Upon the basis of these measurements the director predicts the future angle of elevation of the airplane ε_f by means of a relation which in a simplified form may be approximately written

$$\varepsilon_f = \varepsilon_p + t\,\omega_\varepsilon \quad * \tag{1}$$

in which t is the time of flight.

* This formula neglects the variation in ω_ε as ε changes.

Let the probable error in the measurement of S be ΔS (see figure 2) then it follows that the probable error in the future slant range, $\Delta R = \Delta S \frac{R}{S}$. If it is assumed that present angle of elevation ε_p is measured accurately it follows from (1) that $\Delta\varepsilon_f$, the probable error in the future angle of elevation is equal to $t\Delta\omega_\varepsilon$ in which $\Delta\omega_\varepsilon$ is the probable error in the measurement of ω_ε.*

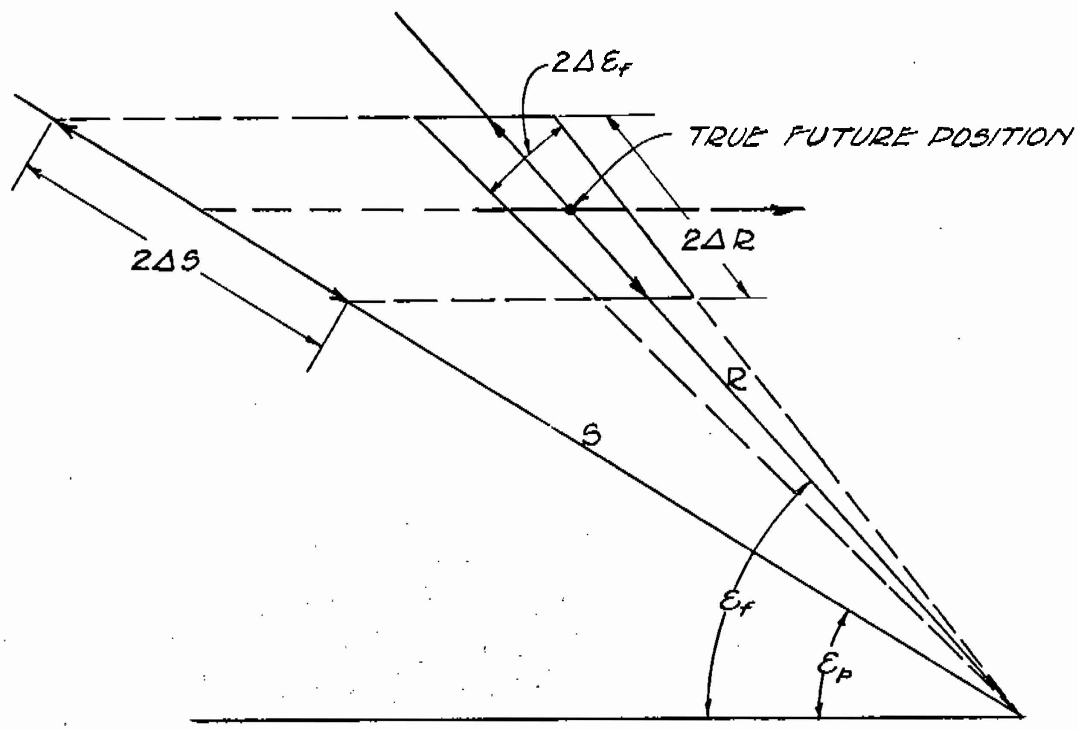

Fig. 2

From the definition of ΔR, it follows that the probability is $\frac{1}{2}$ that predicted slant range will lie within $\pm\ \Delta R$ of the true slant range. Likewise the probability is $\frac{1}{2}$ that predicted angle of elevation will be within $\pm\ \Delta\varepsilon_f$ of the true future angle of elevation. If these probabilities are assumed independent, then the probability is $\frac{1}{4}$ that the predicted position will lie within the quadrilateral of figure 2 the approximate area of which is

* This assumes that t is correctly given.

$$2\Delta R \times 2R\Delta \varepsilon_f = 2\Delta S \frac{R}{S} 2Rt\Delta\omega_\varepsilon = \frac{4\Delta S\ R^2}{S} t\Delta\omega_\varepsilon .$$

Let us now consider the measurement of the azimuth angle α. If α_f is the future value of the azimuth and $\Delta\alpha_f$ is the probable error, we assume that $\Delta\alpha_f = t\Delta\omega_\alpha$, in which $\Delta\omega_\alpha$ is the probable error in the measurement of ω_α, the angular rate in azimuth.

It follows then that the probability is $\frac{1}{8}$ that the predicted position will lie within a volume (see figure 3) having the true future position as center and having dimensions

$$2\Delta R \times 2Rt\Delta\omega_\varepsilon \times 2Rt\Delta\omega_\alpha \cos\varepsilon_f = \frac{8\Delta S}{S} R^3 t^2 \Delta\omega_\varepsilon \Delta\omega_\alpha \cos\varepsilon_f .$$

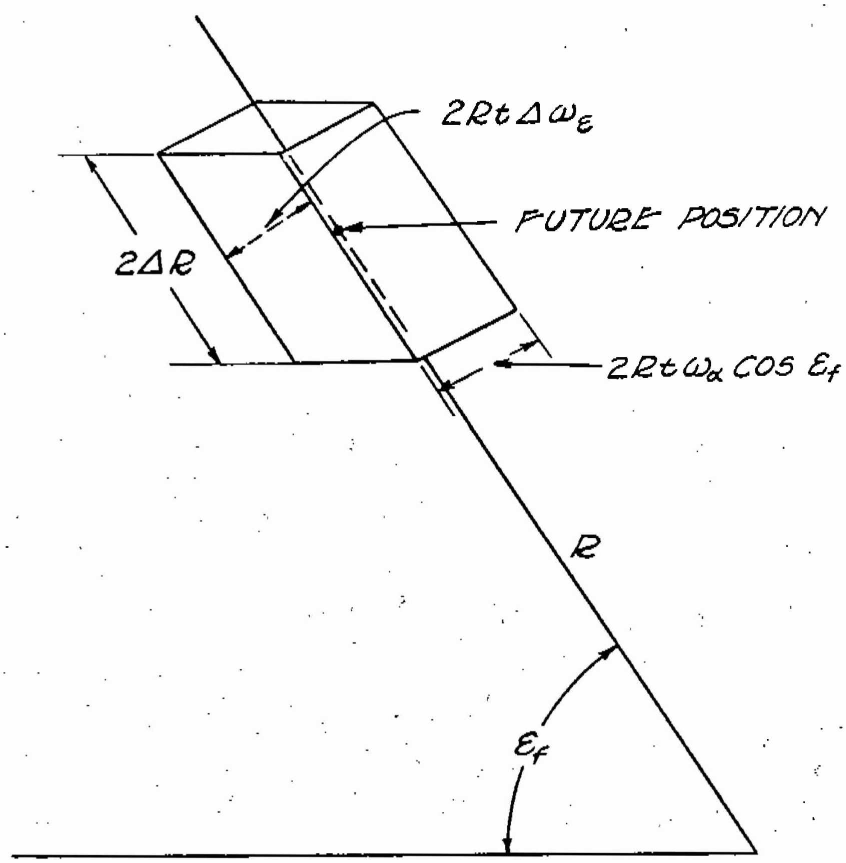

Fig. 3. 12-1/2% volume for the future position.

The matter of the actual dimensions of the volume shown in figure 3 is of great importance. We shall consider three possibilities.

(1) If all three dimensions of the volume of figure 3 are small compared with dispersion in range, elevation and deflection of the ammunition, then there will be no advantage in reducing the volume. Under these conditions then, assuming rectilinear flight with constant speed and altitude, the probability of hitting the airplane would be practically independent of R, t and the $\Delta\omega$'s.

(2) Suppose that $Rt\Delta\omega_\varepsilon$ and $Rt\Delta\omega_\alpha \cos\varepsilon_f$ are negligible in comparison with the dispersion of the ammunition but that ΔR is not. Then the probability of hitting will depend only upon ΔR and will be independent of t and the $\Delta\omega$'s.*

(3) Finally suppose that all three dimensions ΔR, $Rt\Delta\omega_\varepsilon$, $Rt\Delta\omega_\alpha \cos\varepsilon_f$ are large compared with the corresponding components of the dispersion. Under these conditions, the probability of hitting will vary inversely as

$$\frac{\Delta S}{S} R^3 t^2 \Delta\omega_\varepsilon \Delta\omega_\alpha \cos\varepsilon_f.$$

Under these conditions the effectiveness of fire will depend actually upon the magnitude of R, t, and the $\Delta\omega$'s.

We proceed to examine the data obtained in tests of fire control apparatus to see which, if any, of the hypotheses stated above hold for fire against modern aircraft.

It may easily be shown on theoretical grounds that ΔS should vary approximately as S^2. In other words,

$$\Delta S = a_h S^2$$

and

$$\Delta R = \frac{\Delta S \, R}{S} = a_h SR$$

where a_h is a constant depending on the height finder and the operator.

Values of the constant a_h for various operators may be obtained from the curves given on plot 1 which is a reproduction of figure 20 of Accuracy Test, A.A. Height and

* Except for the fact that ΔR depends upon S which in turn depends upon t as well as upon R.

Position Finder Dec. 15, 1935, O.P. 5107 by T. F. Colleran. From these curves it is found that an average value of a_h for the different operators and the most accurate instrument, the T16 is 3.1×10^{-6} 1/yd.

At a slant range as low as 5000 yds. we find $\Delta S = 3.1 \times 10^{-6} \times 5000^2 = 77$ yds. It is apparent that even at this short slant range, ΔS is considerably greater than the probable error of the ammunition in slant range*, and for greater ranges the discrepancy will be even greater. Thus the hypothesis 1 is untenable.

We proceed to obtain values for $\Delta \omega_\varepsilon$ and $\Delta \omega_\alpha$ to determine whether hypothesis 2 or hypothesis 3 holds most nearly.

Data on these may be obtained from the Final Report, Accuracy Test of A.A. Fire Control Instruments, A.P.G. Nov. 1st 1932. This report does not give values of $\Delta \omega_\varepsilon$ and $\Delta \omega_\alpha$ but rather values of $\Delta \varepsilon_f$ and $\Delta \alpha_f$. According to the report algebraic mean values of $\Delta \varepsilon_f$ and $\Delta \alpha_f$ are taken as measurements of the accuracy of the directors. This method of judging the instruments seems to be incorrect. One might have a zero mean error with all the individual errors large. Under these circumstances of course no hits would be secured.

Exhibit A of the report gives the absolute errors in ε_f and α_f from which, t being known, the absolute errors in $\Delta\omega_\varepsilon$ and $\Delta \omega_\alpha$ may be computed** Such computations have been made with the results shown on plots 2 - 10. These plots do not represent all the data given in Exhibit A of the Report on A.A. Fire Control Instruments but they are sufficiently exhaustive, it is believed, to give a representative picture of the errors of the service directors.

In making these plots all observations in which $\Delta \varepsilon_f$ or $\Delta \alpha_f$ exceed 50 mils were rejected and also for some of the plots, observations at the beginning of a course because the operator seemed not be have steadied down.

The results of plots 2 - 10 inclusive are summarized on plot 11. This shows the mean absolute error in angular

* The probable error in slant range at a slant range of 15,000 yds. and time of flight of 30 sec. should be less than 75 yds. and correspondingly less at shorter slant ranges.

** By means of the approximate assumption that $\Delta\varepsilon_f = t\Delta\omega_\varepsilon$ etc.

rate obtained from a group of points having approximately the same angular rate plotted against the corresponding mean angular rate. From plot 1, it is seen that on the whole the mean error in angular rate is approximately proportional to the angular rate, and that there seems to be no systematic distinction between coming and crossing courses.

Assuming that the error is proportional to the rate, the mean value of the ratio, $\frac{\text{mean absolute error in rate}}{\text{angular rate}}$ was computed for each director and multiplied by .845 to give the ratio $\frac{\Delta\omega}{\omega}$. The results are given below.

Director	$\Delta\omega/\omega$
R. A. Corr.*	.097
T-8	.099
M1	.101
M1A1	.128
M2	.166

Suppose an airplane is flying at an altitude of 10,000 yds. at a speed of 150 yds. a second. Assume furthermore that the present slant range S is 18,250 yd., the future slant range R, 15,000 yd, the time of flight 30 sec and that the course of the airplane makes an angle of 30° with the vertical plane including the director and the airplane. Under these conditions

$$\omega_\varepsilon = \frac{150 \cos 30° \ \frac{10,000}{18,250}}{18,250} ^{**} \text{ radians/sec}$$

and if $\frac{\Delta\omega}{\omega} = .1$, $\Delta\omega_\varepsilon = .00039$ radians/sec. If this is multiplied by R and t to give the probable error in vertical deflection in yards, we find

$$Rt\Delta\,\omega_\varepsilon = 175 \text{ yds.}$$

For the probable error in horizontal deflection $Rt\Delta\,\omega_\alpha \cos \varepsilon_f$, we find a value of 220 yds. Both of these errors are large com-

* In these computations, the R.A. Corrector was not penalized for dead time.

**The predicted position is based on the present angular rates. Thus the present angular rates should be used in computing ω_ε and ω_α.

pared with the corresponding errors of the ammunition which should be less than 50 yd. Thus for the conditions assumed hypothesis (3) holds and the probability of hitting varies inversely as

$$a_h\ SR^3 t^2 \Delta\ \omega_\varepsilon \Delta\, \omega_\alpha \cos \varepsilon_f = a_h a_d^2\ SR^3 t^2 \omega_\varepsilon\, \omega_\alpha \cos \varepsilon_f$$

where a_d is a constant representing $\frac{\Delta\omega}{\omega}$. As we have seen for the best service directors a_d is approximately .1.

For these conditions, the value of ΔR the probable error in predicted slant range is found to be

$$3.1 \times 10^{-6}\ SR = 3.1 \times 10^{-6} \times 18{,}250 \times 15{,}000 = 850 \text{ yds.}$$

While at these large slant ranges ΔR is considerably greater than $Rt\Delta\omega = a_d Rt\,\omega$, ΔR varies as SR while $a_d\ Rt\omega$ varies approximately as t since for a given speed of the airplane. $R\omega$ remains approximately constant. Hence at short ranges, the director errors will be greater than the error of the height finder. Moreover, the error of the height finder involves only one dimension while the errors of the director involves two dimensions except for special conditions, such as occur when either $R\omega_\varepsilon$ or $R\omega_\alpha \cos\varepsilon_f$ are small.

If it is assumed that a hit will be obtained when the point of burst is placed in a box having dimensions a, b, c, suitably placed with respect to the airplane, it may be shown that the probability of a hit is

$$\frac{.477^3\ a\ b\ c}{\pi^{3/2} a_h a_d^2\ SR^3 t^2 \omega_\varepsilon \omega_\alpha \cos \varepsilon_f} = \frac{.0195\ a\ b\ c}{a_h a_d^2\ SR^3 t^2 \omega_\varepsilon\, \omega_\alpha \cos \varepsilon_f}\text{*}. \qquad (2)$$

* This result is obtained by the extension of the results of p. 452 of Hayes' Elements of Ordnance to three dimensions. This formula is based on the following assumptions.

(1) $a_h SR$, $a_d Rt\,\omega_\varepsilon$, $a_d Rt\omega_\alpha \cos\,\varepsilon_f$ are large compared with the corresponding components of the ammunition.

(2) They are also large compared with the corresponding dimensions of the box, a, b, c.

(3) $\Delta\varepsilon_f = t\Delta\omega_\varepsilon$, $\Delta\alpha_f = t\Delta\omega_\alpha$.

Assumptions (1) and (2) with the present fire control instruments and a plane speed of 300 mi/hr should hold approximately at slant ranges over 5000 yds except for courses, in which ω_ε or $\omega_\alpha \cos \varepsilon_f$ are small. At the shorter ranges the formula overestimates the probability of hitting.

As a result of the inaccuracy of assumptions (3), the formula may be in error in some cases by as much as 30%.

If we take a = b = c = 50 yds and apply the formula to the situation discussed in the preceding we find that the probability of a hit is $\frac{1}{11,300}$ if $a_h = 3.1\times10^{-6}$ and $a_d = .10$.

It is interesting to see how the probability of hitting varies as the altitude and speed of the airplane change. If a plane flies at a representative azimuth α and elevation ε, R and S will vary approximately as Y the altitude and ω_ε and $\omega_\alpha \cos \varepsilon_f$ will vary as $\frac{v}{Y}$, where v is the speed of the airplane. Accordingly the probability of hitting will vary inversely as

$$Y^4 t^2 \frac{v^2}{Y^2} = Y^2 t^2 v^2.$$

By the aid of this we can find how much the probability of hitting will increase if in the problem discussed above the altitude is reduced from 10,000 yds. to 5000 yds. the time of flight from 30 to 15 sec and the speed of the airplane, v, from 150 yd/sec to 50 yd/sec. These changes it is obvious will augment the probability of hitting by a factor of $2^2 \cdot 2^2 \cdot 3^2 = 144$. In other words the probability of a hit will increase from $\frac{1}{11300}$ to $\frac{144}{11300} = \frac{1}{78}$.

Returning now to the more nearly accurate expression (2) we note that at a given slant range R, S may be changed only slightly; to augment appreciably the probability of hitting one must either reduce t the time of flight, or $\Delta\omega$ or both.

In view of the large errors, $\Delta\omega$, obtained with the service directors, it is evident that for the attack of modern airplanes at the ranges considered and even for much shorter ones, attempts should be made to procure new directors having much smaller errors of angular rate, $\Delta\omega$, than the directors now in service. There is urgent need also of reducing the error of the height finder, ΔR. The director errors are probably more serious than the height finder error however, since the director errors affect in general two dimensions, vertical and lateral deflection while the height finder error affects only one dimension, slant range.

For the directors to be fairly satisfactory for directing fire against modern airplanes their errors must be reduced by a factor of the order of magnitude of 5. To

obtain such a reduction it appears likely that an entirely new type of instrument will be needed, possibly one which does not depend upon manual operation alone to obtain the angular rates ω_ε and ω_α.

For satisfactory results at slant ranges of 15,000 yd the error of the height finder should be reduced by a factor of approximately 10.

Acknowledgments

The computations for the plots were made by Mr. Holberton. Valuable suggestions were obtained from Dr. Dederick and Mr. Colleran.

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Addendum to Ballistic Research Laboratory Report 127.
The Probability of Hitting an Airplane as Dependent
upon Errors in the Height Finder and the Director

Effect of Errors in the Time of Flight on $\Delta\varepsilon_f$ and $\Delta\alpha_f$.

Revision of Formula to Represent more Accurately the Fragmentation Characteristics of the Projectile.

Effect of errors in the time of flight on $\Delta\varepsilon_f$ and $\Delta\alpha_f$.

As mentioned on page 3 of this report the assumption that $\Delta\varepsilon_f = t\Delta\omega_\varepsilon$ and $\Delta\alpha_f = t\Delta\omega_f$ implies that t is correctly given. If we take ω_ε to be constant we have $\varepsilon_f = t\omega_\varepsilon + \varepsilon_p$ (see figure 2 of the report).

If the error in ε_p is considered negligible, then by the well known law of the propagation of error

$$\Delta\varepsilon_f = \left[\left(\frac{\partial\varepsilon_f}{\partial t}\,\Delta t\right)^2 + \left(\frac{\partial\varepsilon_f}{\partial\omega_\varepsilon}\,\Delta\omega_\varepsilon\right)^2\right]^{\frac{1}{2}} =$$

$$\left[(\omega_\varepsilon\,\Delta t)^2 + (t\Delta\omega_\varepsilon)^2\right]^{\frac{1}{2}} \qquad (3)$$

in which Δt represents the probable error in t resulting from the error of the height finder, ΔR.

If we take $\Delta\omega_\varepsilon = a_d\omega_\varepsilon$, (see page 8) then (3) may be written

$$\Delta\varepsilon_f = \omega_\varepsilon\left[a_d^2t^2 + (\Delta t)^2\right]^{\frac{1}{2}}. \qquad (3a)$$

The time of flight, t, depends mainly on the slant range, R, hence, approximately

$$\Delta t = \frac{\partial t}{\partial R}\,\Delta R.$$

From page 5, we have

$$\Delta R = a_h \, SR,$$

from which

$$\Delta t = a_h \frac{\partial t}{\partial R} SR.$$

If this is substituted in (3a) it becomes

$$\Delta \varepsilon_f = \omega_\varepsilon \left[a_d^2 \, t^2 + \left(a_h \frac{\partial t}{\partial R} SR\right)^2 \right]^{\frac{1}{2}} = a_d \, \omega_\varepsilon \left[t^2 + \left(\frac{a_h}{a_d} \frac{\partial t}{\partial R} SR\right)^2 \right]^{\frac{1}{2}}. \quad (3b)$$

In a similar manner, it may be shown that

$$\Delta \alpha_f = a_d \omega_\alpha \left[t^2 + \left(\frac{a_h}{a_d} \frac{\partial t}{\partial R} SR\right)^2 \right]^{\frac{1}{2}}.$$

The product $\Delta\varepsilon_f \; \Delta\alpha_f$ is given by

$$\Delta \varepsilon_f \; \Delta \alpha_f = a_d^2 \, \omega_\varepsilon \omega_\alpha \left[t^2 + \left(\frac{a_h}{a_d} \frac{\partial t}{\partial R} SR\right)^2 \right]^{\frac{1}{2}}.$$

From this result it is obvious that to correct the expression (2) on page 8 to allow for the error in the time of flight due to the error in the slant range as given by the height finder,

$$t^2 + \left(\frac{a_h}{a_d} \frac{\partial t}{\partial R} SR\right)^2$$

must be substituted for t^2 in (2). In this way we obtain as the probability that a burst will occur within a box of dimensions a, b, c.

$$\frac{.0195 \; a \; b \; c}{a_h a_d^2 \; SR^3 \left[t^2 + \left(\frac{a_h}{a_d} \frac{\partial t}{\partial R} SR\right)^2 \right] \omega_\varepsilon \omega_\alpha \cos\varepsilon_f} \quad (2a)$$

Expression (2a) may also be written as

$$\frac{.0195\ a\ b\ c}{a_h\ SR^3\left[a_d^2 t^2 + (a_h\ \frac{\partial t}{\partial R}\ SR)^2\right]\ \omega_\varepsilon \omega_\alpha\ \cos f} \qquad (2b)$$

Unless $(a_h \frac{\partial t}{\partial R} SR)^2$ is appreciably less than $a_d^2\ t^2$, it is apparent that no great increase in the probability of hitting will result from improvements in the director, i.e., reduction of magnitude of a_d. If $a_h = 3.1 \times 10^{-6}$ as given by the present height finder, $t = 30$, $R = 15{,}000$ and $\frac{\partial t}{\partial R} = \frac{1}{280}$ (approximately the reciprocal of the remaining velocity at the target in yd/sec if the curvature of the trajectory is small) then

$$a_2^2\ t^2 \text{ and } (a_h \frac{\partial t}{\partial R}\ SR)^2$$

are practically equal. Hence, under these conditions if the director error were reduced to zero, the probability of hitting would only be doubled.. At slant ranges of 7500 yds. with $t = 15$ and higher remaining velocities the value of

$$(a_h \frac{\partial t}{\partial R}\ SR)^2$$

would be less than 1/4 that of $a_d^2 t^2$. Under these conditions a considerable reduction in the director error would produce a greater increase in the probability of hitting.

Revision of formula (2a) to represent more accurately the fragmentation characteristics of a projectile.

Formula (2a) assumes in effect that an airplane will be disabled if the burst occurs within a box of dimensions a, b, c, surrounding the airplane and that the airplane will not be disabled if the burst occurs outside of the box. Objections to this assumption are given in Ballistic Research Laboratory Report No. 132,"The Probability of Hitting Various Parts of an Airplane as Dependent on the Fragmentation Characteristics of a Projectile". In the report it is

shown that the probability, p_A of hitting a small target of projected area A with at least one fragment is given by

$$p_A = \frac{.0195\ N\ A\ r_e}{a_h a_d^2\ SR^3 \left[t^2 + (\frac{a_h}{a_d}\ \frac{\partial t}{\partial R}\ SR)^2\right] \omega_\varepsilon \omega_\alpha \cos \varepsilon_f} \tag{4}$$

In this, N is the number of effective fragments and r_e the average range of the effective fragments, i.e., the length of the trajectory in which the fragments remain effective. From this report we reproduce some numerical results based on (4) below. It is believed that the probabilities deduced from (4) give a more reliable estimate of the effectiveness of fire with the existing fire control equipment than the probabilities given on page 9 of this report.

Numerical results for p_A for various conditions

We calculate the probability that the pilot with A=1 will be hit by a fragment for various conditions using equation (4b) of which the f is given by the co-factor of a b c in (1a) under various conditions with $a_h = 3.1 \times 10^{-6}$, $a_d = .1$ as follows:

Altitude yd.	Present Slant range yd.	Air Speed yd/sec	Course	Projectile	Time of Flight	N**	r_e✝	p_A
10000	18250	150	30°coming	3"Mk IX	30*	429	134	$\frac{1}{49,000}$
10000	18250	150	30°coming	105 mm MI	30	980	170	$\frac{1}{17,000}$
5000	9100	50	30° "	3" Mk IX	15	429	80	$\frac{1}{280}$
5000	9100	50	30° "	105 mm MI	15	980	100	$\frac{1}{100}$

* This would require a very great muzzle velocity.

** Assuming the fragments, caught on the #3 screen are effective.

✝ These ranges make allowance for the reduced air density. See foot note on page 11 (of Report 132).

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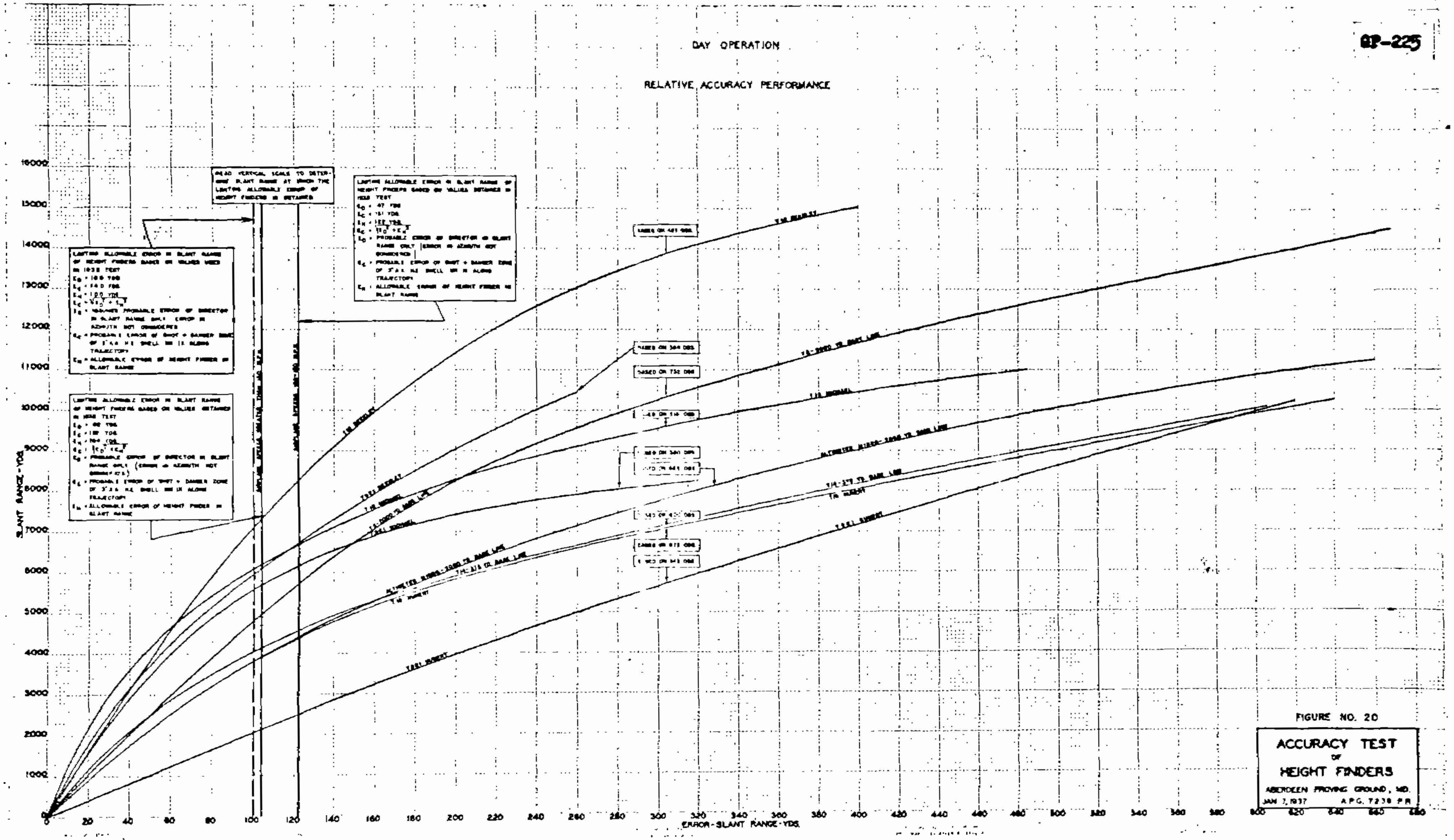
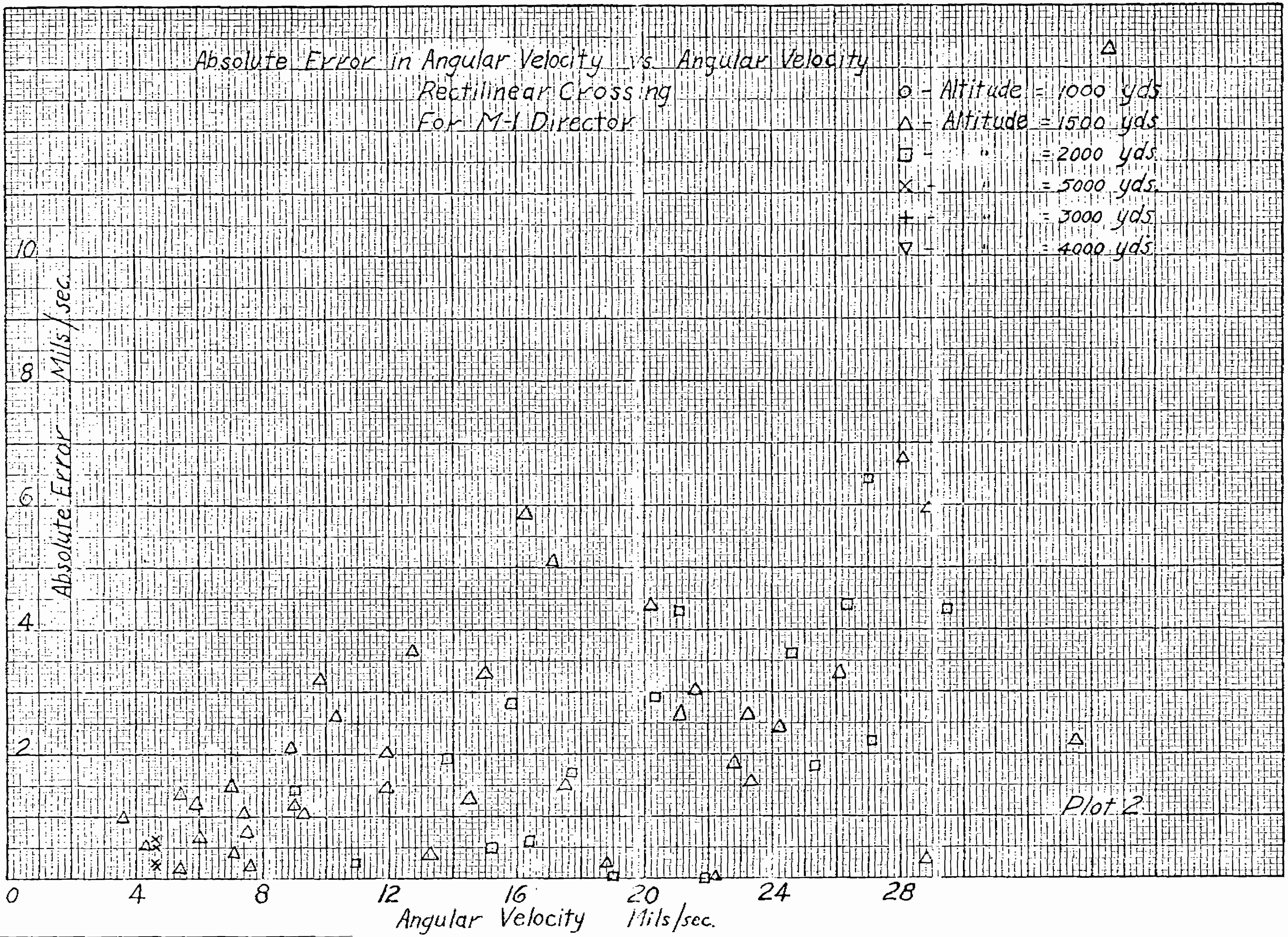
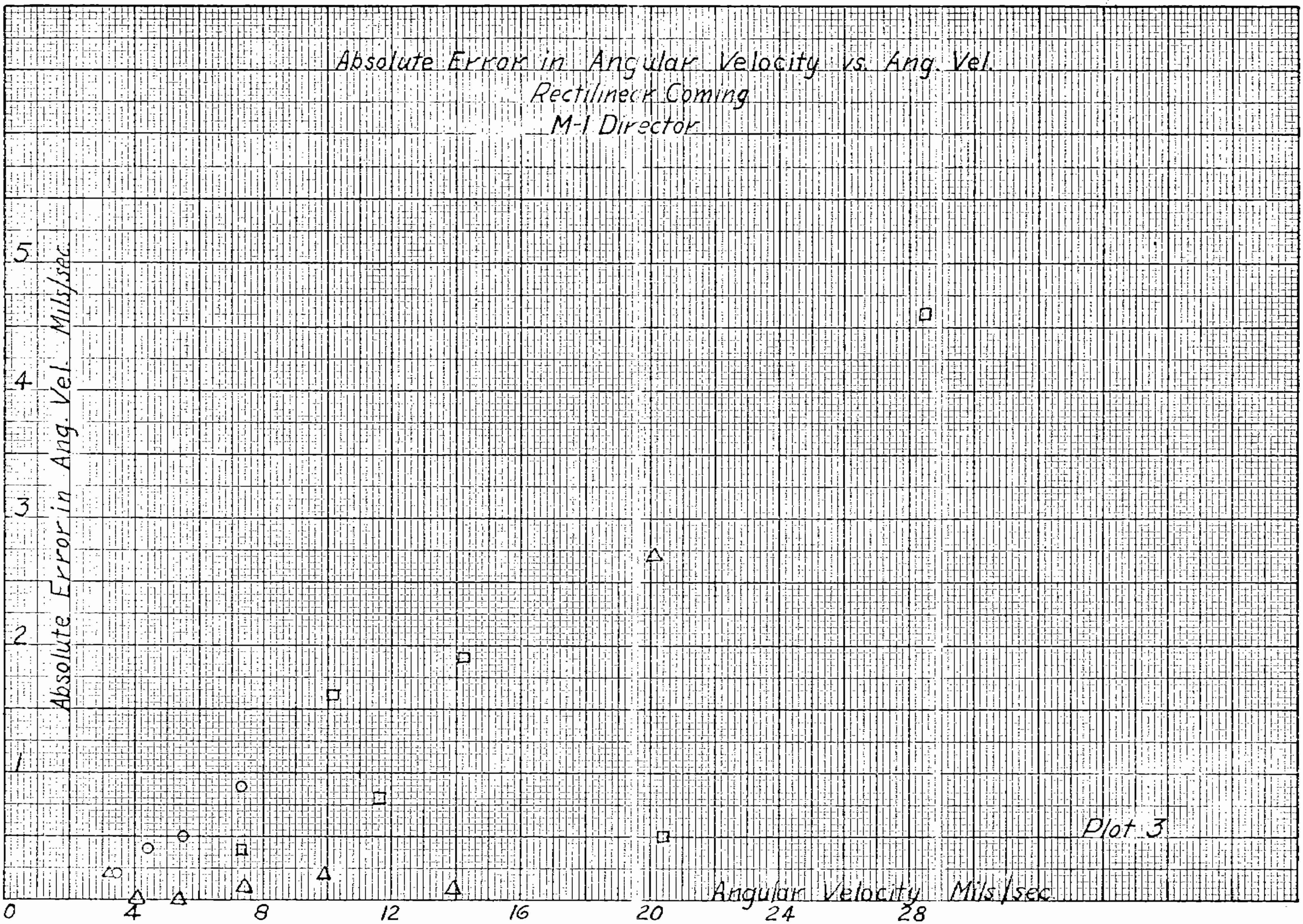
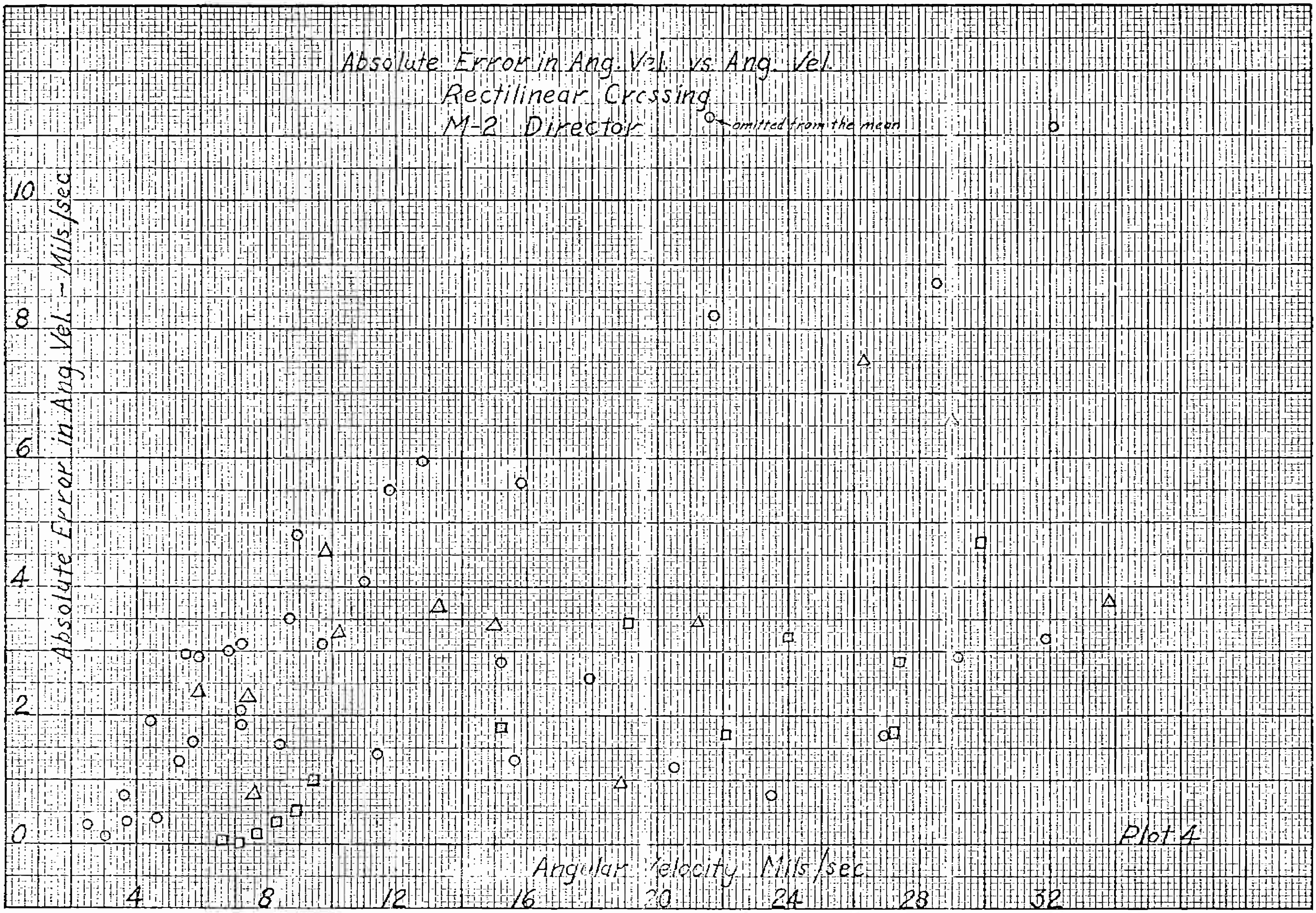


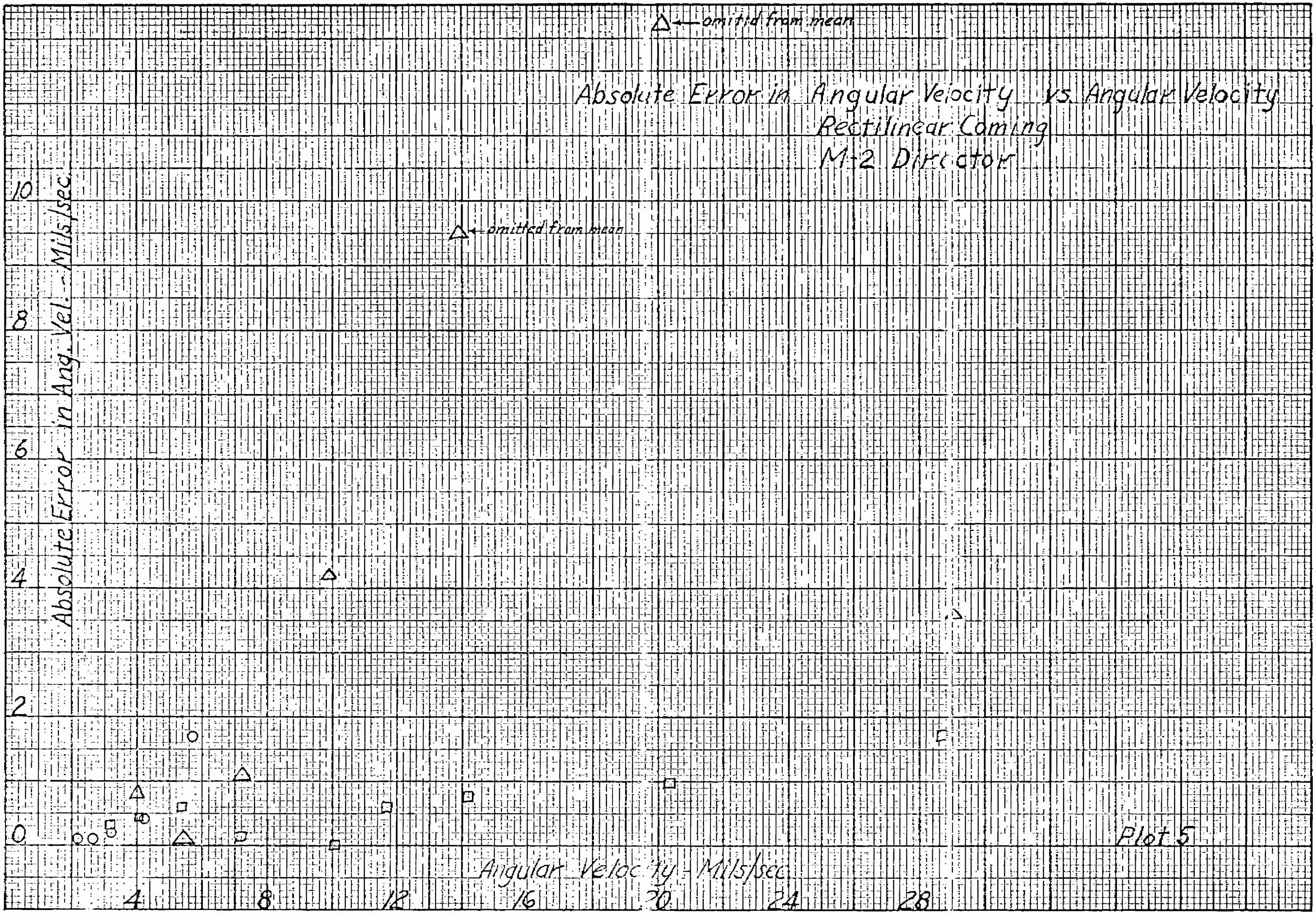
Plot 1

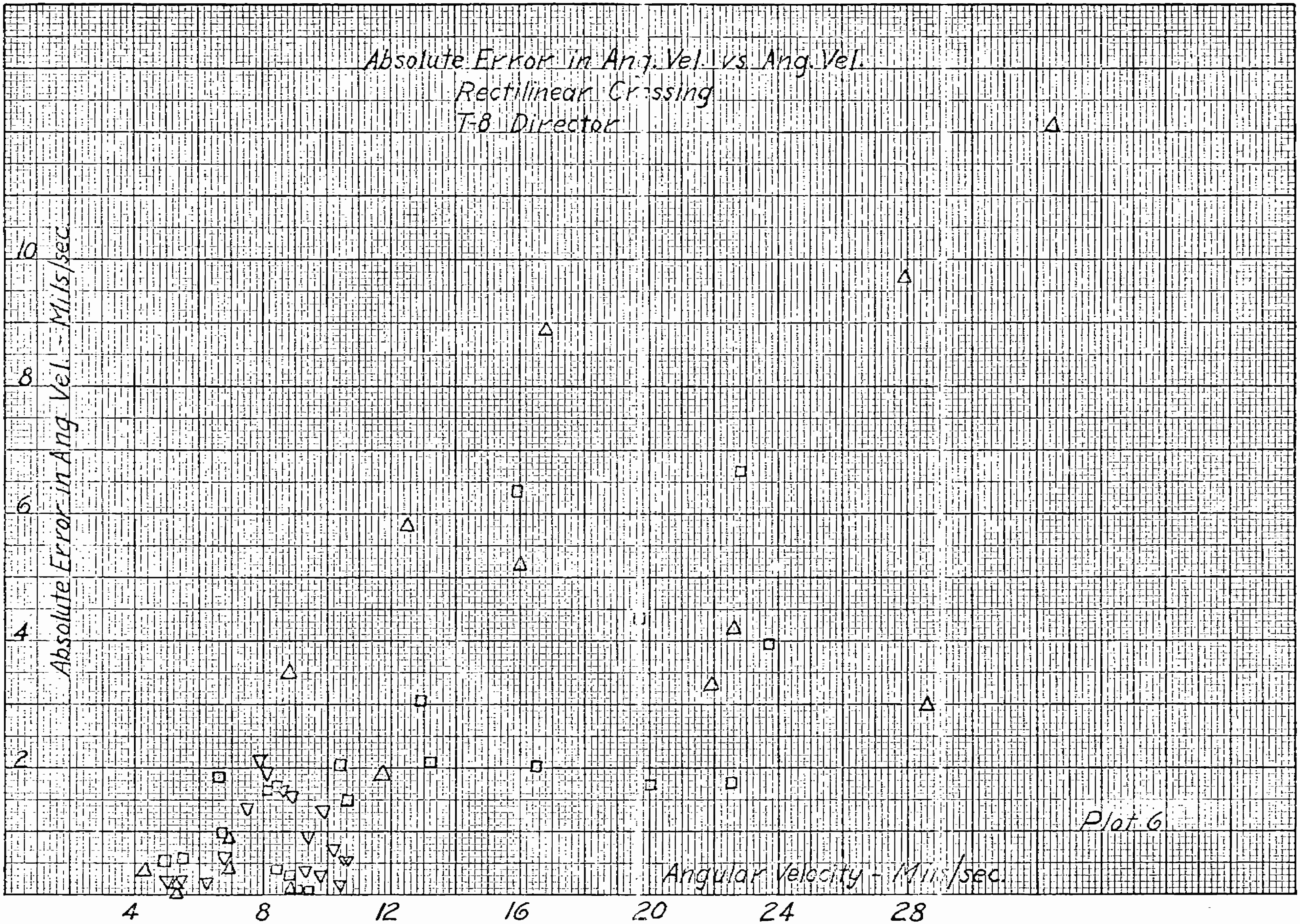
15


Absolute Error in Angular Velocity vs. Angular Velocity
Rectilinear Crossing
For M-1 Director
o - Altitude = 1000 yds
△ - Altitude = 1500 yds
□ - " = 2000 yds
× - " = 5000 yds.
+ - " = 3000 yds
▽ - " = 4000 yds
Absolute Error Mils/sec.
Angular Velocity Mils/sec.
0
2
4
6
8
10
12
16
20
24
28
Plot 2

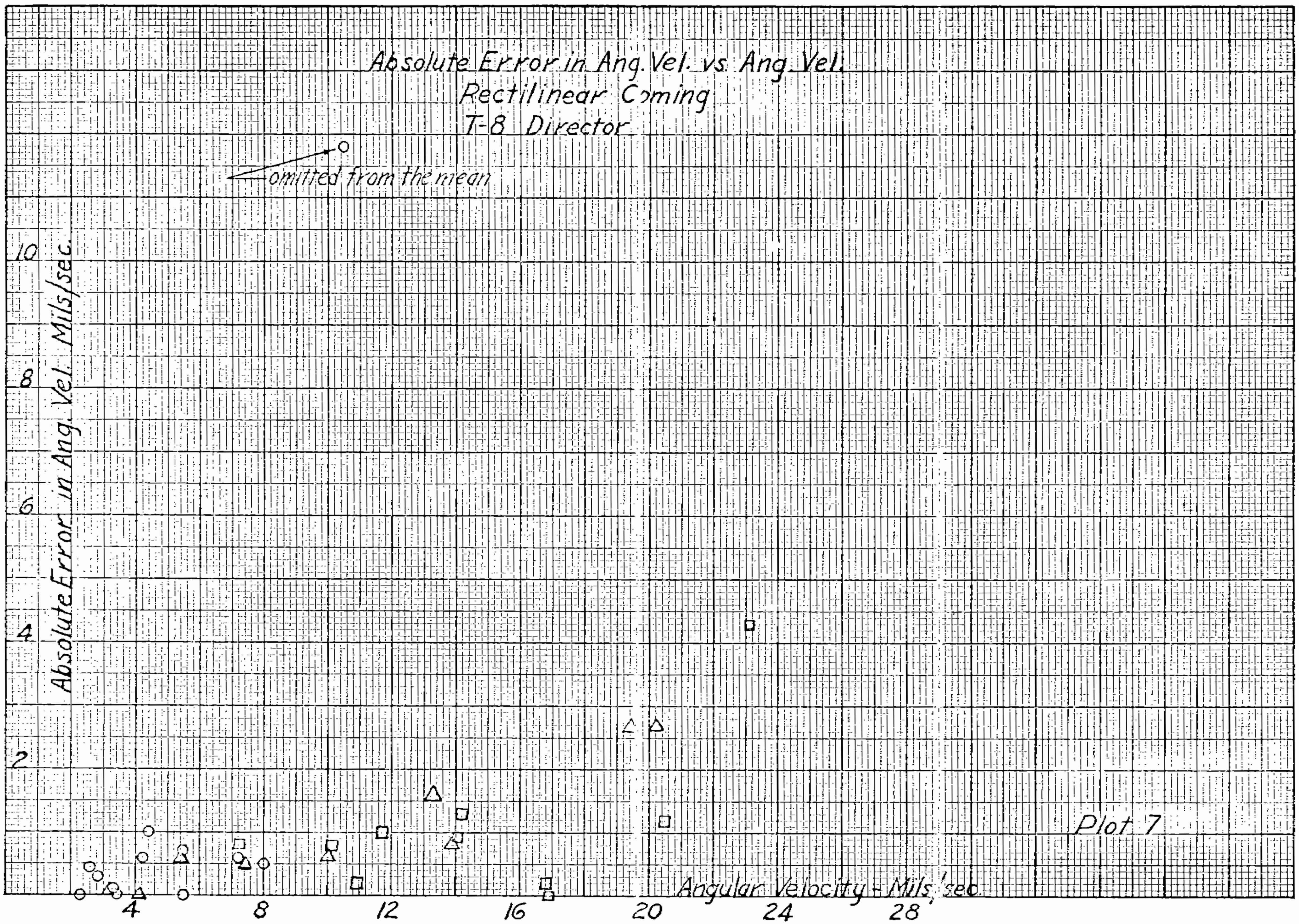

Absolute Error in Angular Velocity vs. Ang. Vel.
Rectilineck Coming
M-1 Director
Absolute Error in Ang. Vel. Mils/sec
5
4
3
2
1
0
4
8
12
16
20
24
28
Angular Velocity Mils/sec
Plot 3

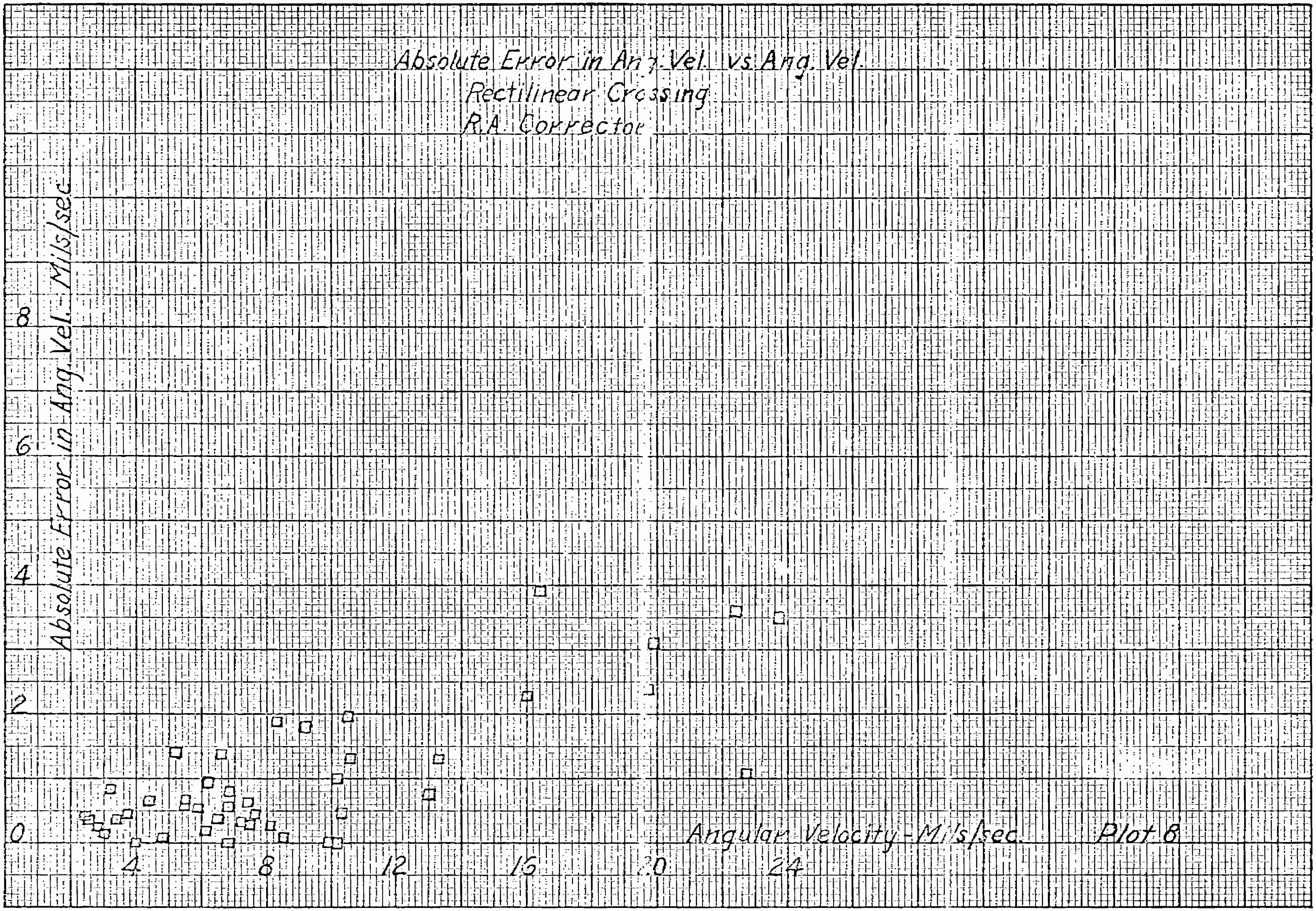

Absolute Error in Ang. Vel. vs Ang. Vel.
Rectilinear Crossing
M-2 Director
omitted from the mean
Absolute Error in Ang. Vel. - Mils/sec
10
8
6
4
2
0
Angular Velocity Mils/sec
4
8
12
16
20
24
28
32
Plot 4


Absolute Error in Angular Velocity vs Angular Velocity
Rectilinear Coming
M-2 Director
omitted from mean
omitted from mean
Absolute Error in Ang. Vel. - Mils/sec.
Angular Velocity - Mils/sec
Plot 5
0
2
4
6
8
10
4
8
12
16
20
24
28


Absolute Error in Ang. Vel. vs Ang. Vel.
Rectilinear Crossing
T-8 Director
Absolute Error in Ang. Vel. - Mils/sec.
10
8
6
4
2
Angular Velocity - Mils/sec.
4
8
12
16
20
24
28
Plot 6

20


Absolute Error in Ang. Vel. vs Ang. Vel.
Rectilinear Coming
T-8 Director
omitted from the mean
Absolute Error in Ang. Vel. Mils/sec.
10
8
6
4
2
4
8
12
16
20
24
28
Angular Velocity - Mils/sec.
Plot 7


Absolute Error in Ang. Vel. vs. Ang. Vel.
Rectilinear Crossing
R.A. Corrector
Absolute Error in Ang. Vel. - Mils/sec.
8
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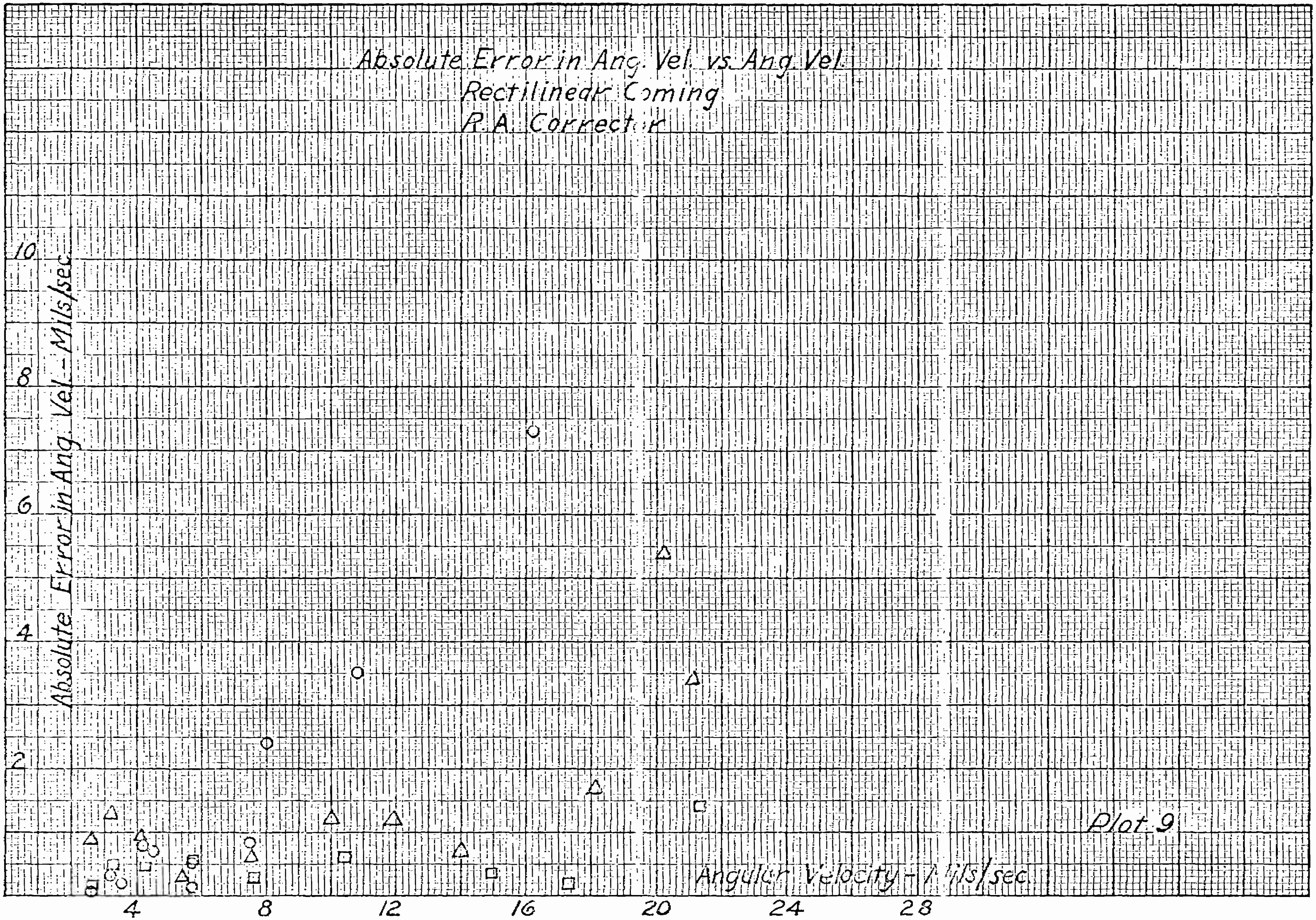
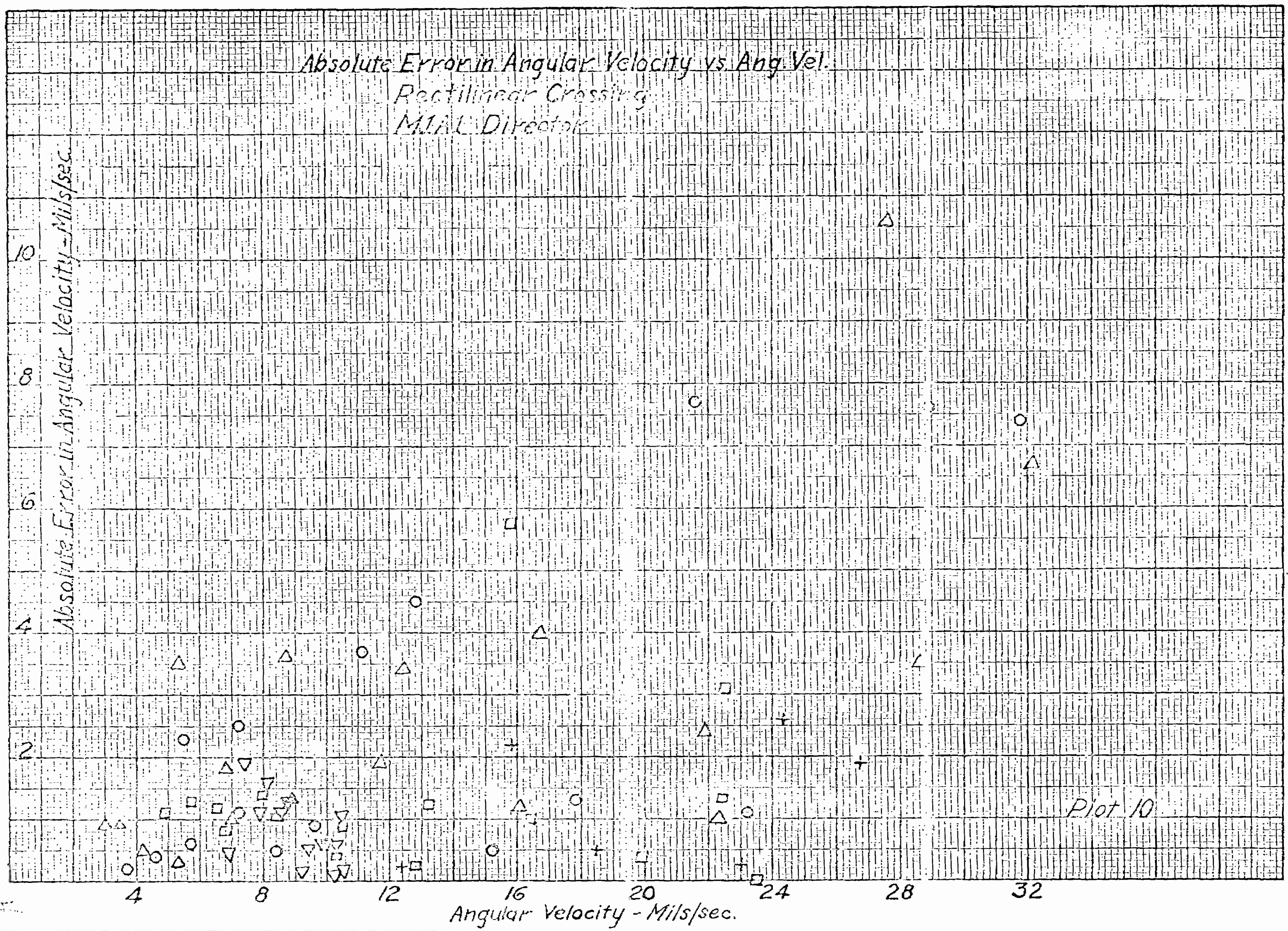
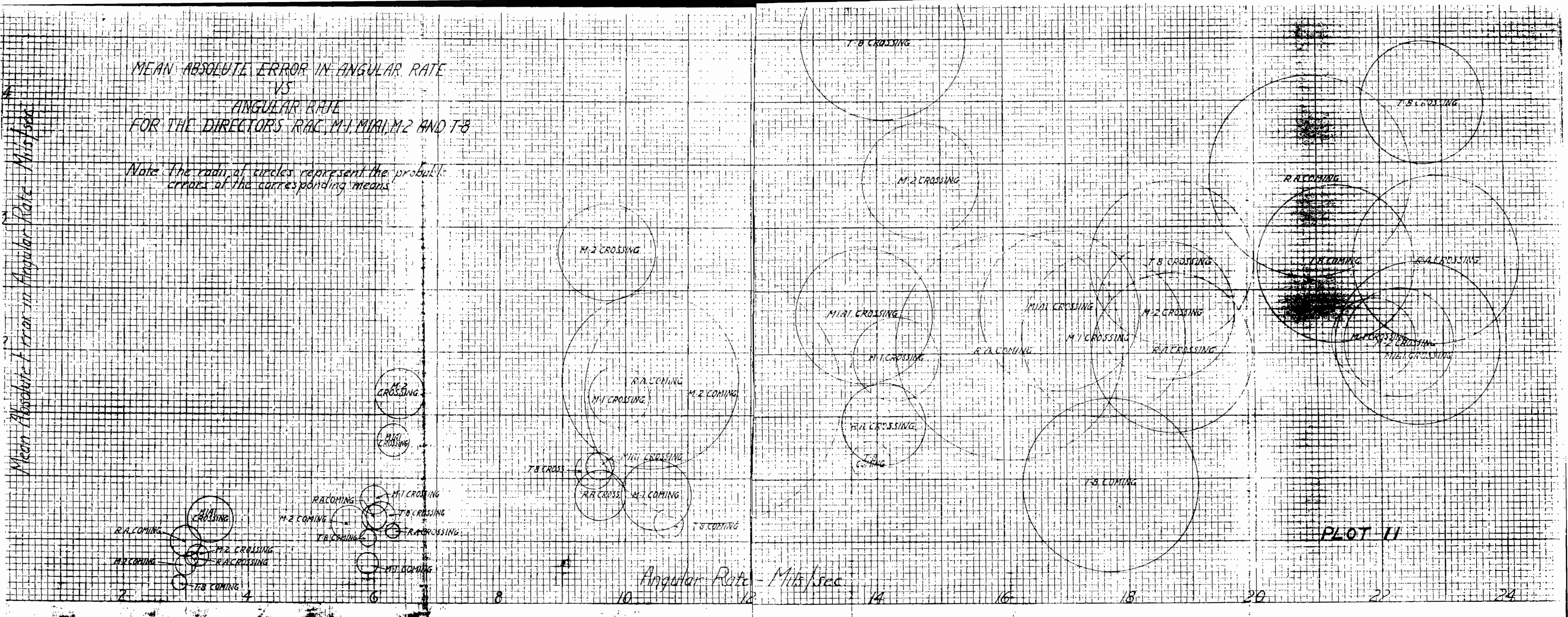
2
0
Angular Velocity - Mils/sec.
4
8
12
16
20
24
Plot 8


Absolute Error in Ang. Vel. vs Ang. Vel.
Rectilinear Coming
R.A. Corrector
10
8
6
4
2
Absolute Error in Ang. Vel. - Mils/sec
4
8
12
16
20
24
28
Angular Velocity - Mils/sec
Plot 9


Absolute Error in Angular Velocity vs Ang Vel.
Rectilinear Crossing
M1A1 Director
Absolute Error in Angular Velocity - Mils/sec.
10
8
6
4
2
4
8
12
16
20
24
28
32
Angular Velocity - Mils/sec.
Plot 10



MEAN ABSOLUTE ERROR IN ANGULAR RATE
VS
ANGULAR RATE
FOR THE DIRECTORS RAC, M.1, M1A1, M.2 AND T-8
Note: The radii of circles represent the probable errors of the corresponding means
Mean Absolute Error in Angular Rate - Mils/sec
Angular Rate - Mils/sec
2
4
6
8
10
12
14
16
18
20
22
24
T-8 CROSSING
M.2 CROSSING
M1A1 CROSSING
M.1 CROSSING
R.A. CROSSING
T-8 COMING
R.A. COMING
M.2 COMING
M.1 COMING
PLOT 11

TITLE: The Probability of Hitting an Airplane As Dependent upon Errors in the Height Finder and the Director AUTHOR(S): Kent, R. H. ORIGINATING AGENCY: Aberdeen Proving Ground, Ballistic Research Lab., Md. PUBLISHED BY: (Same)	ATI- 36083 REVISION (None) ORIG. AGENCY NO. BRL-127 PUBLISHING AGENCY NO. (Same)

DATE	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
Dec '38	Unclass.	U.S.	Eng.	24	diagrs, graphs

ABSTRACT:

It is shown that the probability is 1/8 that the predicted future position of the airplane will lie in a volume surrounding the true future position and having a content which varies as the square of the time of flight and the product of the errors in the two angular rates provided by the director. From an analysis of the performance of service directors it is found that for fire against modern planes at high altitudes, the dimensions of this volume are large compared with the corresponding components of the dispersion of the points of burst and that therefore the probability of hitting varies inversely as this volume. It is pointed out that there is an urgent need for directors of greater precision to augment the probability of hitting as well as for more accurate height finders.

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